Schwinger- Dyson Equations and Dynamical Symmetry Breaking in Quantum R^2- gravity

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The dymamical chiral symmetry breaking in higher- derivative quantum gravity has been investigated on the flat background. The Schwinger-Dyson equations numerical solutions have been found in the ladder approximation. Both two- and four- dimensional cases have been considered. The dymamical fermion mass generation accompanied by the second- order phase transition has been shown to take place in a different gauges.

*Supported by Deutscher Akademischer Austauschdienst, e-mail: shilnov@itp.uni-leipzig.de. Permanent e-mail: shilnov@express.kharkov.ua.

1. Introduction.

The dynamical symmetry breaking (DSB) has been considered as a possible mechanism of appropriable fermion mass generation in the quantum field theory for a long time [1- 20]. Usually, the broken symmetry is the chiral one, which plays a very important role in the high energy physics [3- 12, 18, 19]. There are some methods of DSB investigation: the direct calculation of composite fields effective action [1, 6, 7, 16], Schwinger- Dyson equations (SDE) analysis [3- 8, 13, 15, 17, 18] or solution of composite field renormalizations group equations [20].

These methods have been generalized for curved spacetime case in 4D QED [21], four- fermionic models [22- 28], Einstein general relativity [29] and 2D induced gravity [30, 31]. The results of the DSB in curved spacetime have been the subject of considerable attention because the curvature induced phase transitions, accompanied by the creation of non- zero vacuum expectation values of both elementary [32, 33] and composite bosonic [33- 35] or fermionic [21- 33] fields, is turned out to be important to construct the realistic Early Universe scenario.

It is well-known that, unfortunately, the self-consistent quantum theory of gravity does not exist now. Because the Einstein general relativity is unrenormalizable [36- 37], we have to find out the other models with more attractive quantum features. The most natural and simplest generalization of Einstein theory is the R^2- gravity [33, 38- 44][1]. It is renormalizable and asymptotically free theory. However, it has some disadvantages, such as the non-unitarity in the usual pertubative theory sense and unphysical ghosts presence. From the modern point of view the higher- derivative gravity is nothing but the model including the next term in the low enrgy expansion of future complete quantum theory, based perhaps, on (super)string theory. This opinion lets us don't take into account the last problems because we anyway have to work below the Planck scale, naturally limiting the low energy gravity physics [33].

Therefore, the investigation of the DSB both in 2D induced in the (super)string theory R^2-gravity [45- 48] and in the 4D original version of this model provides us some essential information about the possible features of future complete quantum theory of gravity.

[1] For the review of recent results, see [33].

In the present paper we apply SDE formalism to quantum R^2 gravity with the fermions on the flat background. The ladder or rainbow approximation, when the vertex of fermion- graviton interaction and graviton propagator are taken to be free is used. Both Landau- like general covariant and conformal gauges are considered in 2D case. The covariant gauge providing the minimal structure of graviton Green function (GF) is chosen for 4D spacetime. SDE are obtained and the integral equations determing the exact fermion GF are written evidently. The numerical analysis of their solutions is done in details. The DSB is shown to exist in a different gauges. The dependance of dynamical fermion mass on the coupling constant is found out.

2. Dynamical symmetry breaking in two dimensional $\mathbf{R}^2$- gravity.

We will consider here the theory with the following action:

$$S = \int d^2x\sqrt{-g}\Big[\frac{1}{2M^2}(R^2+2\Lambda) + i\overline{\psi}\gamma^\mu(x)D_\mu\psi\Big], \tag{1}$$

where R is the space-time curvature, ψ- 2D spinors, Λ- cosmological constant and the dimensional value M is the fermion– graviton coupling constant.

$$D_\mu = \partial_\mu + \frac{1}{2}\omega_\mu^{ab}\sigma_{ab} \tag{2}$$

is the spinor covariant derivative with the standard spin- connection ω_μ^{ab}.

Local Dirac matrices $\gamma_\mu(x)$ can be expressed through the usual flat ones γ_a and tetrads $e_a^\mu(x)$: $\gamma^\mu(x) = e_a^\mu(x)\gamma^a$ and, finally, $\sigma_{ab} = \frac{1}{4}[\gamma_a,\gamma_b]$. Greek and Latin indices correspond to the curved and flat tangent spacetimes respectively.

In accordace with the general background field method one gets:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{3}$$

We choose the gauge- fixing term in the folowing form [49] :

$$S_{gf} = \frac{-\beta_1}{2M^2}\int d^2x\sqrt{-g}(\nabla_\lambda h_\mu^\lambda - \beta_2\nabla_\mu h)(\eta^{\mu\nu}\nabla_\rho\nabla^\rho + \beta_3\nabla^\mu\nabla^\nu)(\nabla_\sigma h_\nu^\sigma - \beta_2\nabla_\nu h), \tag{4}$$

where $h = h^\mu_\mu$ and $\beta_1, \beta_2, \beta_3$ – gauge- fixing dimensionless parameters. The most useful tool for the calculation of the graviton Green functions for these kinds of complicate expressions is the projective operators method [50]. It gives

$$G^{\mu\nu\rho\sigma}(k) = M^2\Big[\Big(-\frac{4\beta_1(\beta_2-1/2)^2(1+\beta_3)+1}{\delta}+\frac{4}{(\Lambda-\beta_1 k^4)k^4}\Big)k^\mu k^\nu k^\rho k^\sigma +$$
$$+\frac{-\beta_1 k^2}{\Lambda(\Lambda-\beta_1 k^4)}\Big(\eta^{\mu\rho}k^\nu k^\sigma+\eta^{\mu\sigma}k^\nu k^\rho+\eta^{\nu\rho}k^\mu k^\sigma+\eta^{\nu\sigma}k^\mu k^\rho\Big)+$$
$$+\Big(\frac{2\beta_1(\beta_2-1)(\beta_2-1/2)(1+\beta_3)}{\delta}-\frac{2}{\Lambda k^4}\Big)\Big(\eta^{\mu\nu}k^\rho k^\sigma+\eta^{\rho\sigma}k^\mu k^\nu\Big)k^2+$$
$$+\Big(\frac{2}{\Lambda}+\frac{\Lambda}{4\delta}-\frac{\beta_1(\beta_2-1)^2(1+\beta_3)k^4}{\delta}\Big)\eta^{\mu\nu}\eta^{\rho\sigma}-\frac{1}{\Lambda}\Big(\eta^{\mu\rho}\eta^{\nu\sigma}+\eta^{\mu\sigma}\eta^{\nu\rho}\Big)\Big], \quad (5)$$

where

$$\delta = -\beta_1(\beta_2-1)^2(1+\beta_3)k^8+\frac{\Lambda k^4}{4}\Big(1+4\beta_1(\beta_2-1/2)^2(\beta_3+1)\Big). \quad (6)$$

Graviton - fermion interaction vertex has the usual form [29, 30]:

$$\Gamma_{\mu\nu}(p,k) = \frac{1}{8}(2p^\lambda+k^\lambda)\gamma^\sigma(2\eta_{\lambda\sigma}\eta_{\mu\nu}-\eta_{\lambda\mu}\eta_{\sigma\nu}-\eta_{\lambda\nu}\eta_{\sigma\mu}). \quad (7)$$

The most general Lorentz invariant form for the inverse exact fermion GF is the following:

$$S^{-1}(p) = A(p^2)\not p - B(p^2), \quad (8)$$

where $A(p^2)$ and $B(p^2)$ are the unknown functions we should find out.

The SDE for GF (8) in the ladder approximation are given by:

$$S^{-1} - S_0^{-1}(p) = \int \frac{d^2q}{(2\pi)^2 i}\Gamma_{\mu\nu}(q,p-q)S(q)\Gamma_{\lambda\sigma}(p,q-p)G^{\mu\nu\lambda\sigma}(p-q) \quad (9),$$

where $S_0(p) = 1/\not p$ is the free fermion GF.

These equations determine the extremum of composite fermionic fields effective potential [16]:

$$V_{eff} = -iSp\,[\,ln(S_0^{-1}S) - S_0^{-1}S + 1\,] + V_2, \quad (10)$$

where

$$V_2 = \frac{1}{2}\int \frac{d^2p}{(2\pi)^2}\int \frac{d^2q}{(2\pi)^2} Sp\Big[\Gamma_{\mu\nu}(p-q,q)S(q)\Gamma_{\rho\sigma}(q-p,p)S(q)\Big]G^{\mu\nu\rho\sigma}(p-q) \tag{11}$$

corresponds to the two-particle irreducable vacuum diagrams. Formula (11) is written in the ladder approximation, where the vertices and graviton GF are chosen to be free and the only fermion GF is taken to be exact.

Now we have all the necessary parts of Feynman diagrams to calculate the final expressions for the SDE (9) and the effective potential (10). After the Wick rotation and tedious algebra the integral equations for structure functions $A(x)$ and $B(x)$ are obtained:

$$A(x) = 1 + g\int_0^1 \frac{A(y)dy}{yA^2(y)+B^2(y)}\frac{1}{x}K_A(x,y) \qquad (12),$$

$$B(x) = g\int_0^1 \frac{B(y)dy}{yA^2(y)+B^2(y)}K_B(x,y), \tag{13}$$

and the effective potential (10) is given by

$$V_{eff} = -\frac{T^2}{4\pi}\Big\{\int_0^1 dx\Big[\ln\Big(A^2(x)+\frac{B^2(x)}{x}\Big) - 2\frac{A(x)\Big(A(x)-1\Big)+B^2(x)}{A^2(x)x+B^2(x)}\Big] +$$
$$g\int_0^1 \frac{dx}{A^2(x)x+B^2(x)}\int_0^1 \frac{dy}{A^2(y)y+B^2(y)}\Big[A(x)A(y)K_A(x,y)+$$
$$B(x)B(y)K_B(x,y)\Big]\Big\}, \tag{14}$$

where T is the ultraviolet cut off parameter,

$$g = \frac{1}{64\pi}\frac{M^2}{T^2},\ x = \frac{p^2}{T^2},\ y = \frac{q^2}{T^2},\ A(p^2) = A(x),\ B(x) = \frac{B(p^2)}{T}. \tag{15}$$

The evident form for the integral equations kernels $K_A(x,y)$, $K_B(x,y)$ have been obtained explicitly for the arbitrary β_1, β_2, β_3. However they are too large to present them here.

We would note only that these kernels contain, in general, the terms with unpleasant factors like $(x-y)^{-1}$. It means that the infrared divergencies caused by the virtual massless gravitons whose momentum tends

to zero appear here. This type of divergences arises , probably, because the quantum corrections for the interaction vertex are omitted in our approximation [21, 30].

However in the conformal and Landau- like gauges, which have the most considerable physical meaning, these divergences don't take place [30, 31]. That is why we investigate here the $\beta_1 \to \infty$ case, corresponding to Landau- like gauge. Then

$$K_A(x,y) = \Big[-\frac{\beta_2^2}{(\beta_2-1)^2}\frac{xy+(x+y)(x+y+l_2)/4}{l_2} - \frac{(2\beta_2-1/2)(x-y)^2}{l_1(\beta_2-1/2)}\times$$

$$(x+y+l_2) - \frac{(x-y)^2}{l_1 l_2}((x-y)^2-l_2^2)\Big]\frac{1}{\sqrt{(x+y+l_2)^2-4xy}}+$$

$$+\Big[\frac{\beta_2^2}{(\beta_2-1)^2}\frac{xy+(x+y)(x+y-l_2)/4}{l_2} - \frac{(2\beta_2-1/2)(x-y)^2}{l_1(\beta_2-1/2)}(x+y-l_2)+$$

$$\frac{(x-y)^2}{l_1 l_2}((x-y)^2-l_2^2)\Big]\frac{\tilde{\Theta}(x,y,l_2^{1/2})}{\sqrt{(x+y-l_2)^2-4xy}} + \frac{2\beta_2(x+y)}{l_1(\beta_2-1/2)}|x-y|, \quad (16)$$

$$K_B(x,y) = \Big[\frac{\beta_2^2}{2(\beta_2-1)^2}\Big(1-2\frac{x+y}{l_2}\Big)+\frac{(x-y)^2}{l_1(\beta_2-1/2)}\Big]\frac{\tilde{\Theta}(x,y,l_2^{1/2})}{\sqrt{(x+y-l_2)^2-4xy}}+$$

$$\Big[\frac{\beta_2^2}{2(\beta_2-1)^2}\Big(1+2\frac{x+y}{l_2}\Big)+\frac{(x-y)^2}{l_1(\beta_2-1/2)}\Big]\frac{1}{\sqrt{(x+y+l_2)^2-4xy}}+$$

$$\frac{-2|x-y|}{l_1(\beta_2-1/2)}, \quad (17)$$

where $l_1 = \frac{\Lambda}{T^4}$, $l_2 = \sqrt{l_1}\left|\frac{\beta_2-1/2}{\beta_2-1}\right|$ and

$$\tilde{\Theta}(x,y,a) = \Theta(\,(\sqrt{x}-\sqrt{y})^2 - a^2) - \Theta(a^2-(\sqrt{x}+\sqrt{y})^2). \quad (18)$$

The analytical solution of thes non-linear integral equations does not seem to be possible. Therefore, we present here the results of numerical calculations by means of the standard iterative procedure, described, for example, in [29- 31]

The dependence of structural functions $A(x)$ and $B(x)$ on the Euclidian momentum square is presented in the Fig. 1 for the different values of coupling constant g and fixed $l_1 = 4$ and $\beta_2 = 1.05$. $A(x)$ (curve 1) doesn't

almost depend on g. The only trivial solution $B = 0$ exists for function $B(x)$ for small g. However for $g > g_c = 0.23$ the type of solution changes essentially and only the non-trivilal ones increasing with g growth provide the minimum of effective potential (14). The curves 4, 3, 2 correspond to the following values of $g = 0.25, 0.30, 0.35$ respectively.

The dependence of dynamical mass, defined by the pole of exact fermion GF, on the gauge coupling constant g is shown in the Fig. 2. It means that after the analytical continuation into the pseudoeuclidian region the dynamical mass is obtained as the solution of the equation

$$m^2 A(m^2) - B(m^2) = 0. \tag{19}$$

This Figure shows us clearly the typical behaviour of order parameter m^2 in the course of phase transition accompanied by the creation of bifermionic condensate in the above critical region $g > g_c$.

Let us discuss now the conformal gauge, which plays the very important role in the (super)string theory. In this gauge

$$g_{\mu\nu} = \exp(\varphi)\eta_{\mu\nu}. \tag{20}$$

Then, the graviton propagator is given by:

$$G(k) = \frac{M^2}{k^4 + \Lambda}, \tag{21}$$

and the fermion- graviton interaction vertex are the following [31]:

$$\Gamma(p, k) = \frac{1}{4}\Big(2\not{p} + \not{k}\Big). \tag{22}$$

The equations for the structural functions and and effective potential have the same form as in the previous case (12)- (14) with:

$$\begin{aligned} K_A(x,y) = \frac{x^2 + y^2 + 6xy - l(x+y)}{4l} \frac{\tilde{\Theta}(x,y,l^{1/2})}{\sqrt{(x+y-l)^2 - 4xy}} - \\ \frac{x^2 + y^2 + 6xy + l(x+y)}{4l} \frac{1}{\sqrt{(x+y+l)^2 - 4xy}}, \end{aligned} \tag{23}$$

$$K_B(x,y) = -\frac{2x + 2y - l}{2l} \frac{\tilde{\Theta}(x,y,l^{1/2})}{\sqrt{(x+y-l)^2 - 4xy}} +$$

$$\frac{2x+2y+l}{2l}\frac{1}{\sqrt{(x+y+l)^2-4xy}} \tag{24}$$

where $l=-\frac{\Lambda}{T^4}>0$.

The plot of the functions $A(x)$ and $B(x)$ are presented in the Fig. 3. The dynamical symmetry breaking takes place in this case as well. The behaviour of function $A(x)$ doesn't almost depend on the value g (curve 1). The non-trivial solutions for function $B(x)$ appear for $g>g_c=2.0$ only. The curves 4, 3, 2 correspond to the $g=3,4.5,5.5$. Figure 4 represents the dynamical mass as a function of coupling constant g.

3. Schwinger- Dyson equations in 4D higher- derivative gravity.

The similar program can be realized for the four- dimensional gravity with the Lagrangian, containing square curvature terms (for details, see book [33]):

$$S_g=\int d^4x\sqrt{-g}\Big[\alpha_1R^2+\alpha_2R_{\mu\nu}R^{\mu\nu}+\alpha_3R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}+\Lambda\Big], \tag{25}$$

directly following [33, 49, 50]. $\alpha_1,\alpha_2,\alpha_3$ are the arbitrary constants here.

We choose the parameters of gauge fixing action:

$$S_{gf}=\frac{\beta_1}{2}\int d^4x\sqrt{-g}(\nabla_\lambda h^\lambda_\mu-\beta_2\nabla_\mu h)(\eta^{\mu\nu}\nabla_\rho\nabla^\rho+\beta_3\nabla^\mu\nabla^\nu)(\nabla_\sigma h^\sigma_\nu-\beta_2\nabla_\nu h), \tag{26}$$

demanding that the square on $h_{\mu\nu}$ part of action, $(S+S_{gf})^{(2)}$ won't contain the non-minimal terms. Then:

$$(S_g+S_{gf})^{(2)}=\frac{1}{2}\int d^4x\sqrt{-g}h_{\mu\nu}\Big[\frac{1}{2}\beta_1\beta_2\eta^{\mu\nu}\eta_{\rho\sigma}\Box^2-\frac{1}{2}\beta_1\delta^{\mu\nu}_{\rho\sigma}\Box^2+$$
$$+\frac{\Lambda}{2}(\frac{1}{2}\eta^{\mu\nu}\eta_{\rho\sigma}-\delta^{\mu\nu}_{\rho\sigma})\Big]h^{\rho\sigma}, \tag{27}$$

where

$$\delta^{\mu\nu}_{\rho\sigma}=\frac{1}{2}(\delta^\mu_\rho\delta^\nu_\sigma+\delta^\mu_\sigma\delta^\nu_\rho) \tag{28}$$

and

$$\beta_1=\alpha_2+4\alpha_3,\quad \beta_2=\frac{4\alpha_1+\alpha_2}{4(\alpha_1-\alpha_3)},\quad \beta_3=-\frac{2\alpha_1+\alpha_2+2\alpha_3}{\alpha_2+4\alpha_3} \tag{29}$$

Then, the graviton GF is given by:

$$G^{\mu\nu\rho\sigma} = -\frac{1}{\beta_1(k^4+2\lambda)}\Big[\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} + \frac{2(\beta_2 k^4+\lambda)\eta^{\mu\nu}\eta^{\rho\sigma}}{k^4(1-4\beta_2)-2\lambda}\Big], \qquad (30)$$

where $\lambda = \frac{\Lambda}{2\beta_1}$. The graviton- fermion interaction vertex has the same form as for the covariant gauge in the 2D model (7) and we obtain the same expressions for the SDE (12)- (13) and, with the accuracy up to the positive constant factor, for the effective potentilal (14) but, of course, different kernels:

$$\begin{aligned} K_A(x,y) = \frac{y}{16} + \frac{1}{32x}\Big[&(x+y+l_1-\sqrt{(x+y+l_1)^2-4xy})\\ &\Big(-\frac{l_1^2-xy+7(x+y)^2/4}{2l_1}-\frac{11}{8}(x+y)\Big)+\\ &+(x+y-l_1-\sqrt{(x+y-l_1)^2-4xy}\tilde{\Theta}(x,y,l_1^{1/2}))\times\\ &\Big(\frac{l_1^2-xy+7/4(x+y)^2}{2l_1}-\frac{11}{8}(x+y)\Big)+\\ &\frac{9}{4(1-4\beta_2)}\Big((x+y+l_2-\sqrt{(x+y+l_2)^2-4xy})\times\\ &\Big(\frac{2xy+1/2(x+y)^2}{l_2}-\frac{1}{2}(x+y)\Big)+\\ &\frac{9}{4(1-4\beta_2)}(x+y-l_2-\sqrt{(x+y-l_2)^2-4xy}\tilde{\Theta}(x,y,l_2^{1/2}))\times\\ &\Big(-\frac{2xy+1/2(x+y)^2}{l_2}+\frac{1}{2}(x+y)\Big)\Big], \end{aligned} \qquad (31).$$

$$\begin{aligned} K_B(x,y) = \frac{9}{128 l_1^2 x}\Big[&l_1\Big((2x+2y-l_1)(x+y-l_1-\\ &\sqrt{(x+y-l_1)^2-4xy}\tilde{\Theta}(x,y,l_1^{1/2}))-\\ &(2x+2y+l_1)(x+y+l_1-\sqrt{(x+y+l_1)^2-4xy})\Big)-\\ &l_2\Big((2x+2y-l_2)(x+y-l_2-\sqrt{(x+y-l_2)^2-4xy}\tilde{\Theta}(x,y,l_2^{1/2}))-\\ &(2x+2y+l_2)(x+y+l_2-\sqrt{(x+y+l_2)^2-4xy})\Big)\Big], \end{aligned} \qquad (32)$$

where $l_1^2 = \frac{2\lambda}{T^4}$, $l_2^2 = \frac{l_1^2}{4\beta_2-1}$, $\beta_2 > 1/4$, $g = (16\pi^2\beta_1)^{-1}$.

On the Fig. 5 the plot of functions $A(x)$ (curve 1) and non-trivial solutions of $B(x)$ minimized the effective potential for $g > g_c = 2.5$ (curves 2, 3, 4 corresponds to $g = 4.5, 4.0, 3.4$) are presented. $\beta_2 = 2, l_1 = 0.1$ here. Fig. 6 represents the dymamical mass dependence on the coupling constant g.

4. Conclusions

Thus, the quantum higher-derivative gravity interacted with the fermions has been shown by the numerical analysis to contain the phase with the dynamically broken chiral symmetry and dynamically generated fermionic mass. The most important result of our paper is the fact that existance of such phenomena doesn't depend on the gauge choice in the different models of R^2- gravity. However the values of g_c and the character of phase transition are strongly depend on both the parameters of the model and the gauge type.

We would like to thank S. D. Odintsov for the formulation of the problem and helpful discussions.

We are grateful to S. Falkenberg for the help in the preparation of the manuscript.

Yu. I. Sh. is very much indebted to DAAD for finacial support of his visit to Leipzig University, where the final version of this work has been done and to Prof. B. Geyer for hospitality.

He also expresses his deep gratitude to A. Letwin and R. Patov for their kind support.

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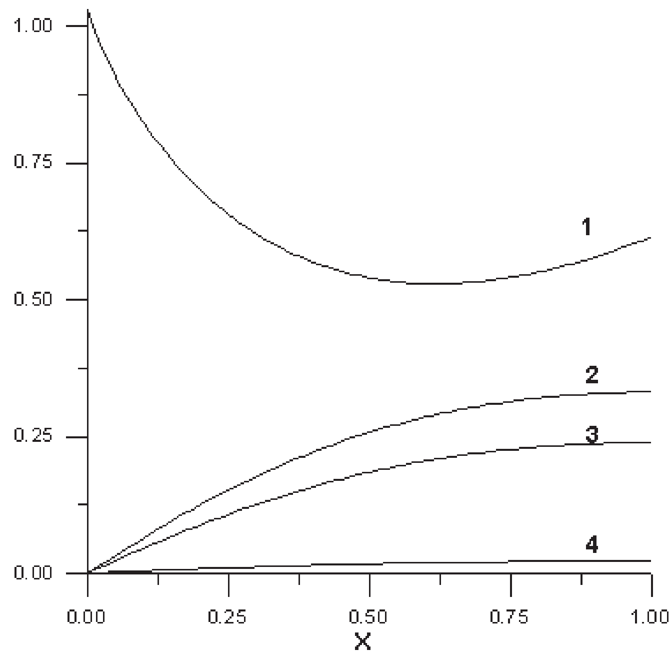

Fig. 1

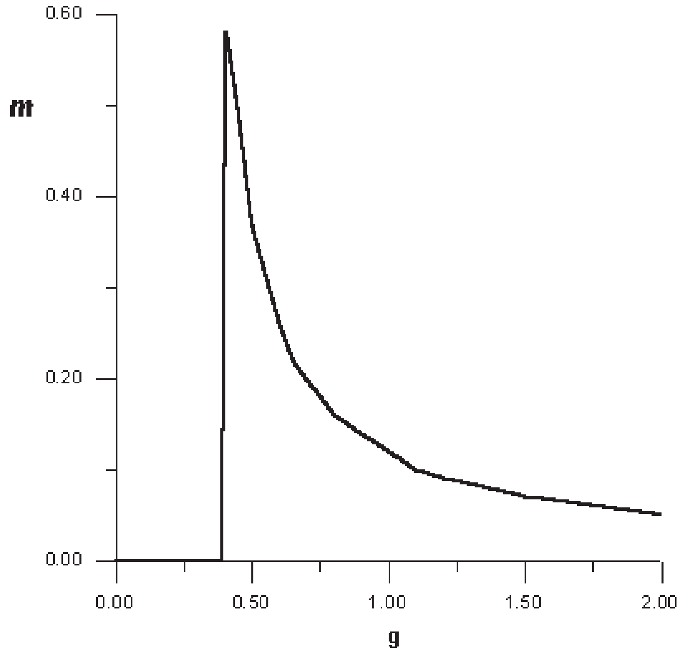

Fig. 2

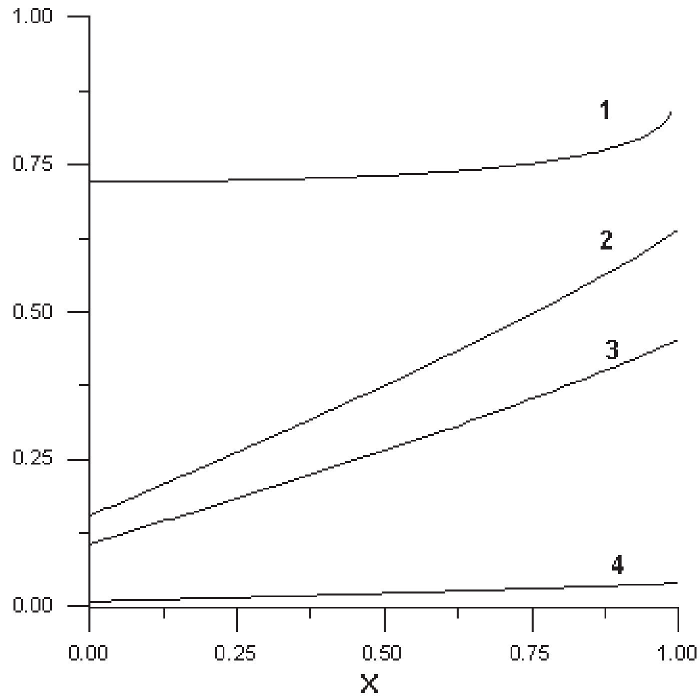

Fig. 3

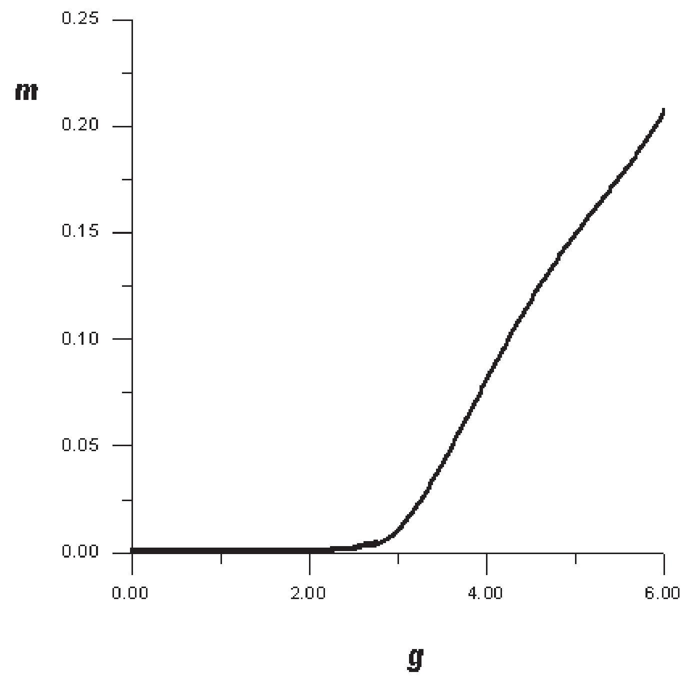

Fig. 4

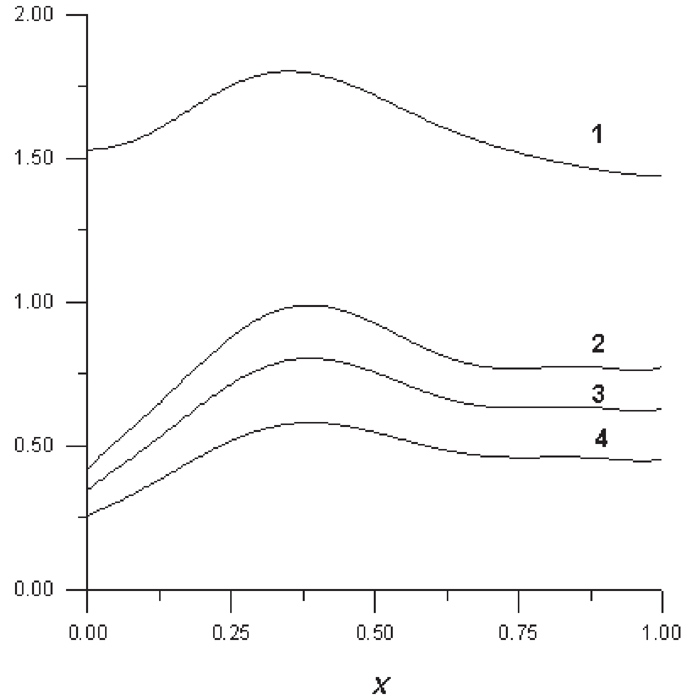

Fig. 5

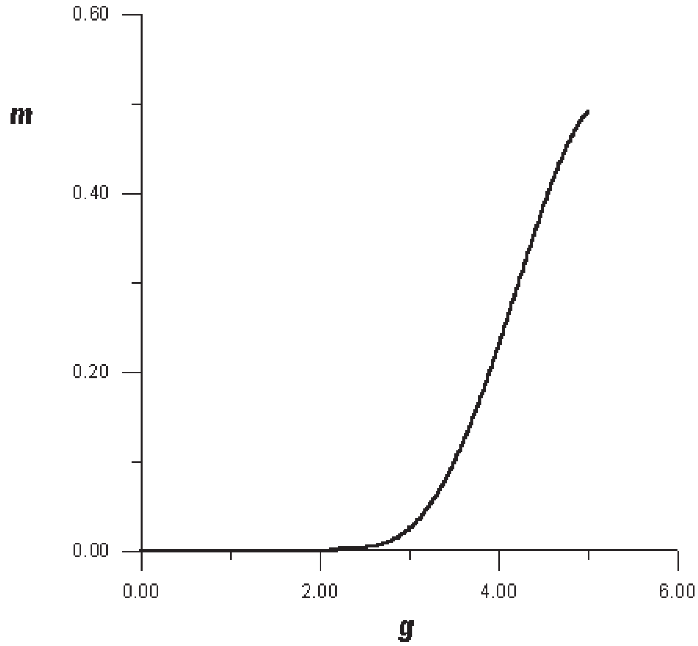

Fig. 6